

# Excitation of multilayered cylindrical structures with the help of ring source

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A cylindrical geometry consisting of a conductor (radius  $a_1$ ), ring source (radius  $a$ ), aircolumn (radius  $b$ ), thin layer of Sb (radius  $c$ ), thin layer of superconductor (radius  $d$ ) and a thin layer of Sb (radius  $e$  such that  $a_1 < a < b < c < d < e$ ) is studied. The ring source is assumed to be the ring of magnetic currents. This structure gives the multiple peaks of radiation field of equal magnitude at different angles including some peaks of less radiation field at other angles for different values of various parameters of this structure

## 1. INTRODUCTION

Source excitation of cylindrical structure has been studied by Sharma *et al* (1975) and Ram Chandra *et al* (1974). In addition to this multilayered transmission lines are also investigated by Dmitriyev *et al* (1973). Here in this article we intend to discuss the radiation pattern of the multilayered system while exciting with the help of ring of magnetic currents. The ring source is assumed to be situated in air. The air column is surrounded by thin layers of antimony, superconductor and antimony. The geometry consists of a conductor along its  $z$  axis. The radius of conductor, ring source, air column, antimony layer, superconductor and outer antimony layer are respectively  $a_1, a, b, c, d$  and  $e$  ( $a_1 < a < b < c < d < e$ ).

## 2. ANALYSIS

The ring source is situated in  $z = 0$  plane. The source term is given by  $M = \hat{\phi} \delta(z) \delta(\rho - a)$  Where  $\hat{\phi}$  is the unit vector along  $\phi$  direction of the cylindrical coordinates and  $\delta$  being the Kronecker's delta function. Following the procedure given by Dhani Ram *et al* (1972) one can arrive at the following solutions for the first asymmetric mode in cylindrical coordinates.

$$h_1 = A_1 J_1(v_0 \rho) + B_1 Y_1(v_0 \rho) \quad a_1 < \rho < a \quad \dots \quad (1a)$$

$$h_2 = A_2 J_1(v_0 \rho) + B_2 Y_1(v_0 \rho) \quad a < \rho < b \quad \dots \quad (1b)$$

$$h_3 = A_3 J_1(v_1 \rho) + B_3 Y_1(v_1 \rho) \quad b < \rho < c \quad \dots \quad (1c)$$

$$h_4 = A_4 J_1(v_2 \rho) + B_4 Y_1(v_2 \rho) \quad c < \rho < d \quad \dots \quad (1d)$$

$$h_5 = A_5 J_1(v_1 \rho) + B_5 Y_1(v_1 \rho) \quad d < \rho < e \quad \dots \quad (1e)$$

$$h_6 = A_6 H_1^{(1)}(v_0 \rho) \quad e < \rho \quad \dots \quad (1f)$$

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Where  $h$  is the Fourier's Transform of  $H_\varphi$  given by

$$h(\rho, k_z) = \int_{-\infty}^{\infty} H_\varphi(\rho, z) e^{jk_z z} dz$$

and the inverse transform is given by

$$H_\varphi(\rho, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} h(\rho, k_z) e^{jk_z z} dk_z$$

$J_n$ ,  $Y_n$ , and  $H_n^{(1)}$  are Bessel functions of first kind, second kind and Hankel function of first kind respectively of  $n$ -th order

$$\begin{aligned} v_0^2 &= w^2 \mu_0 \epsilon_0 - k_z^2 \\ v_1^2 &= w^2 \mu_0 \epsilon_0 \epsilon_a - k_z^2 \\ \text{and} \\ v_2^2 &= w^2 \mu_0 \epsilon_a \epsilon_s - k_z^2 \end{aligned}$$

where  $w$ ,  $\mu_0$ ,  $\epsilon_0$ ,  $\epsilon_a$  and  $\epsilon_s$  are respectively the source frequency, permeability and permittivity of free space, propagation vector, dielectric constant for antenna layer and dielectric constant for superconductor

Now applying the following boundary conditions

- (i) At  $\rho = a_1$ , the tangential electric vector viz  $E_z(\rho, z)$  is continuous. Therefore

$$\left. \frac{dh_1}{d\rho} + \frac{h_1}{\rho} \right|_{\rho=a_1-\delta} = 0$$

- (ii) At the location of the magnetic ring source of electromagnetic waves ( $\rho = a$ ) the continuation of  $H_\varphi$  and jump conditions gives

$$\begin{aligned} h_1|_{a-\delta} &= h_2|_{a+\delta} \\ \left. \frac{dh_2}{d\rho} \right|_{a+\delta} - \left. \frac{dh_1}{d\rho} \right|_{a-\delta} &= -jw\epsilon_p \end{aligned}$$

- (iii) At  $\rho = b$ , the continuation of  $H_\varphi$  and  $E_z$  will give

$$\begin{aligned} h_2|_{b-\delta} &= h_3|_{b+\delta} \\ \epsilon_a \left[ \left. \frac{dh_2}{d\rho} + \frac{h_2}{\rho} \right|_{b-\delta} - \left. \frac{dh_3}{d\rho} + \frac{h_3}{\rho} \right|_{b+\delta} \right] &= 0 \end{aligned}$$

- (iv) At  $\rho = c$ , the continuation of  $H_\varphi$  and  $E_z$  will give

$$h_3|_{c-\delta} = h_4|_{c+\delta}, \quad \frac{\epsilon_s}{\epsilon_a} \left[ \left. \frac{dh_3}{d\rho} + \frac{h_3}{\rho} \right|_{c-\delta} - \left. \frac{dh_4}{d\rho} + \frac{h_4}{\rho} \right|_{c+\delta} \right] = 0$$

(v) Again at  $\rho = d$ , the continuation of  $H_v$  and  $H_z$  will give

$$h_4|_{d+\delta} = h_5|_{d+\delta} = \frac{\epsilon_a}{\epsilon_b} \left[ \frac{dh_4}{d\rho} + \frac{h_4}{\rho} \right]_{d-\delta} = \frac{dh_5}{d\rho} + \frac{h_5}{\rho} \Big|_{d+\delta}$$

(vi) At  $\rho = c$ , the continuation of  $H_v$  and  $H_z$  will give

$$h_5|_{c-\delta} = h_6|_{c+\delta} \left[ \frac{dh_5}{d\rho} + \frac{h_5}{\rho} \right]_{c-\delta} = \epsilon_a \left[ \frac{dh_6}{d\rho} + \frac{h_6}{\rho} \right]_{c+\delta}$$

where  $\delta \rightarrow 0$

After applying these boundary conditions one can get the magnitude of  $A_0$  occurring in eq. (1f)

Now in  $c < \rho$  region

$$H_z = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_3 H_1^{(1)}(H_0 \rho) \exp(jk_z z) dk_z \quad \dots (2)$$

The variables  $v_0$ ,  $v_1$  and  $v_2$  occurring in this eq. have multiple values in the neighbourhood of  $v_0 = 0$ ,  $v_1 = 0$  and  $v_2 = 0$  respectively. The integrand happens to be even function of  $k_z$  when one expands it about  $v_0 = 0$ . But Hankel's function appearing in the integrand has logarithmic singularity and hence gives branch points of the integrand. Considering the eq. (2) to be evaluated in  $k_z$  plane as contour integral. Introduce the transformation

$$k_z = k_0 \sin \tau$$

where  $\tau = \lambda + j\alpha$

This transforms the region of integration in  $k_z$  plane into a strip of  $\tau$  plane, which is bounded by two curved lines corresponding to the branch cuts  $k = \pm k_0$  in  $\tau$  plane. The branch cuts in  $\tau$  plane are given by

$$\sin \lambda \cos h\alpha = \pm 1$$

which yields

$$V_0 = k_0 \cos \tau, \quad V_1 = k_0 \sqrt{\epsilon_a - \sin^2 \tau} \quad \text{and} \\ \Gamma_2 = k_0 \sqrt{\epsilon_s - \sin^2 \tau}$$

and eq. (2) gets transformed to

$$H_z(\rho, z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} A_0(\tau) H_1^{(1)}(k_0 \rho \cos \tau) \exp(jk_0 \sin \tau) k_0 \cos \tau d\tau \quad \dots (3)$$

Shifting from cylindrical coordinates  $(\rho, \phi, z)$  to spherical coordinates  $(r, \theta, \phi)$  and taking approximate value of Hankel function of the integrand of eq. (3) for large values of  $r$ , we have

$$H_0(r, z) \approx \frac{1}{2\pi} \int A_0(\tau) \exp(-3\pi j/4) \left[ \pi k_0 r \cos \theta \cos \tau \right]^{1/2} \exp[jk_0 r \cos(\tau - \theta)] k_0 \cos \tau d\tau \quad \dots (4)$$

Saddle point of eq. (4) is given by

$$\frac{d}{d\tau} \cos(\tau - \theta) = 0$$

which gives  $\tau = \theta$  saddle point.

The steepest Descent Path is given by the constant phase of exponential factor of the integrand of eq. (4) and is to pass through the saddle point. So one can have

$$\text{Im } j \cos(\tau - \theta) = \text{const}$$

and is pass through  $\tau = \theta$  which gives

$$\begin{aligned} \cos(\lambda - \theta) \cosh h\alpha &= 1 \\ \sin(\lambda + \pi/2 - \theta) \cosh h\alpha &= 1 \end{aligned}$$

This is plotted in the Fig. 1. Now taking the help from standard method

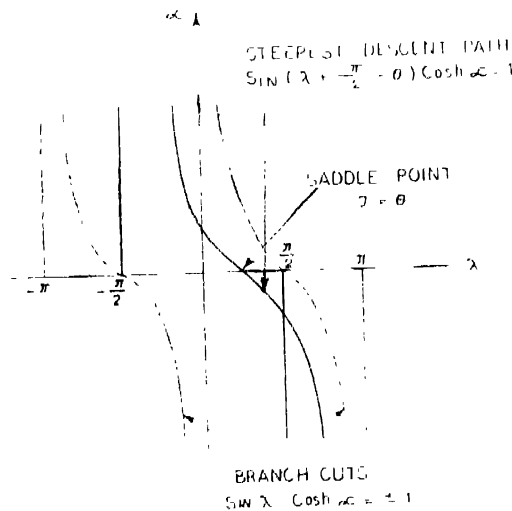


Fig. 1.  $\tau$  plane representation for saddle point method

(Collin, 1960) for the lowest order approximation of the integral, one can end up with

$$H_q(r, \theta) = \frac{J_0^{wa}}{2\pi} F(\theta)^{q-1},$$

Here  $F(\theta) = [A_6(k_2 - k_0 \sin \theta)]^{-1}$  and

$A_6$  is given by

$$A_6 = \frac{R_{73}(R_{78} - R_{76})}{R_{78}R_{75} - R_{77}R_{76}}$$

$$\begin{aligned} R_{71} = & \{ (e_a V_2 Y_1(v_2 d) J_1(v_2 d) - e_a V_2 Y_0(v_2 d) J_1(v_2 d)) \{ Y_1(v_2 d) \} e_a V_2 J_0(v_2 d) Y_1(v_1 c) \\ & J_1(v_2 c) e_s V_1 Y_0(v_1 c) \} \{ J_0(v_1 c) Y_1(v_1 c) - Y_0(v_1 c) J_1(v_1 c) \} e_s V_1 - \{ J_1(v_2 c) e_s V_1 J_0(v_1 c) \\ & e_a V_2 J_0(v_2 d) J_1(v_1 c) \} Y_1(v_2 d) R_{57} - J_1(v_2 d) \{ e_a V_2 Y_0(v_2 c) \} Y_1(v_1 c) \\ & e_s V_1 Y_0(v_1 c) Y_1(v_2 c) \} \{ J_0(v_1 c) Y_1(v_1 c) - Y_0(v_1 c) J_1(v_1 c) \} e_s V_1 \\ & \{ Y_1(v_2 c) e_s V_1 J_0(v_1 c) - e_a V_2 Y_0(v_2 c) J_1(v_1 c) \} R_{57} \} \\ & [e_s V_1 \{ J_0(v_1 c) Y_1(v_1 c) - Y_0(v_1 c) J_1(v_1 c) \} \{ J_1(v_1 c) e_s V_1 Y_0(v_1 c) \\ & - e_a V_2 J_0(v_0 b) Y_1(v_1 c) \} Y_1(v_0 b) ] \left[ \frac{J_1(v_0 d) Y_0(v_0 a)}{J_0(v_0 a)} - \{ V_0 a J_0(v_0 d) - J_1(v_0 d) \} \right. \\ & v_0 a Y_0(v_0 d) \} Y_1(v_0 a) - v_0 a \{ J_0(v_0 d) Y_1(v_0 a) - J_1(v_0 d) Y_0(v_0 a) \} \{ \\ & J_1(v_1 b) v_0 a \{ Y_0(v_0 d) J_1(v_0 a) - Y_1(v_0 d) J_0(v_0 d) \} \{ v_0 a J_0(v_0 d) J_0(v_0 a) \\ & J_1(v_0 a) \} \frac{Y_0(v_0 a)}{J_0(v_0 a)} \} \left. \right] + \{ -e_a V_0 J_0(v_0 b) Y_0(v_0 b) - V_1 J_0(v_1 b) Y_1(v_0 b) \} \\ & J_1(v_0 a) \{ J_0(v_0 a) \} \left. \right] + \{ -e_a V_0 Y_0(v_0 b) \{ J_1(v_0 b) - e_a V_0 J_0(v_0 b) \} \} \\ & [V_0 a Y_1(v_0 b) \{ V_0 a J_0(v_0 a) - J_1(v_0 a) \} \{ J_0(v_0 a) Y_1(v_0 a) - J_1(v_0 a) Y_0(v_0 a) \} \\ & - J_1(v_0 b) V_0 a Y_0(v_0 a) \} \{ Y_0(v_0 a) J_1(v_0 a) - Y_1(v_0 a) J_0(v_0 a) \} \{ V_0 a J_0(v_0 a) - J_1(v_0 a) \} \\ & J_0(v_0 a) \} \} \\ R_{57} = & \{ J_1(v_1 c) e_s V_1 Y_0(v_1 c) - e_a V_0 J_0(v_0 b) Y_1(v_1 c) \} Y_1(v_1 b) \{ \frac{v_0 a J_0(v_0 a)}{J_0(v_0 a)} \\ & \{ Y_0(v_0 d) J_1(v_0 a) - Y_1(v_0 d) J_0(v_0 d) \} \{ v_0 a J_0(v_0 a) - J_1(v_0 a) \} \} \\ & J_1(v_1 b) v_0 a \{ Y_0(v_0 a) \} \{ Y_0(v_0 d) J_1(v_0 a) - Y_1(v_0 d) J_0(v_0 d) \} \\ & \{ v_0 a J_0(v_0 a) - J_1(v_0 a) \} \left. \right] + \{ e_a V_0 J_1(v_1 b) Y_0(v_0 b) - V_1 J_0(v_1 b) Y_1(v_0 b) \} \\ & [v_0 a J_0(v_0 a) - J_1(v_0 a) \} \left. \right] + \{ e_a V_0 Y_0(v_0 b) \{ J_1(v_0 b) - e_a V_0 J_0(v_0 b) \} \} \\ & [v_0 a Y_1(v_0 b) \{ v_0 a J_0(v_0 a) - J_1(v_0 a) \} \{ J_0(v_0 a) Y_1(v_0 a) - J_1(v_0 a) Y_0(v_0 a) \} \\ & - J_1(v_0 b) v_0 a \{ Y_0(v_0 a) \} \{ Y_0(v_0 d) J_1(v_0 a) - Y_1(v_0 d) J_0(v_0 d) \} \{ v_0 a J_0(v_0 a) - J_1(v_0 a) \} \} \\ & J_0(v_0 a) \} \} \\ R_{72} = & \{ \{ e_s V_1 J_0(v_1 d) J_1(v_2 d) - e_a V_2 J_1(v_1 d) J_1(v_2 d) \} \{ Y_1(v_2 d) R_{59} - J_1(v_2 d) R_{60} \} \\ & + R_{59} J_1(v_1 d) \{ Y_1(v_2 d) e_a V_2 J_1(v_2 d) - e_a V_2 Y_0(v_2 d) J_1(v_2 d) \} \} \} \end{aligned}$$

$$\begin{aligned}
 R_{59} &= \{e_a V_2 J_0(v_2 c) Y_1(v_1 c) - J_1(v_2 c) e_s V_{-1} Y_0(v_1 c)\} \{J_0(v_1 c) Y_1(v_1 c) \\
 &\quad - Y_0(v_1 c) J_1(v_1 c)\} e_s V_{-1} - \{J_1(v_2 c) e_s V_{-1} J_0(v_1 c) - e_a V_{-2} J_0(v_2 c) J_1(v_1 c)\} R_{57} \\
 R_{60} &= \{e_a V_2 Y_0(v_2 c) Y_1(v_1 c) - e_s V_{-1} Y_0(v_1 c) Y_1(v_2 c)\} \{J_0(v_1 c) Y_1(v_1 c) \\
 &\quad - Y_0(v_1 c) J_1(v_1 c)\} e_s V_{-1} - \{Y_1(v_2 c) e_s V_{-1} J_0(v_1 c) - e_a V_{-1} Y_0(v_2 c) J_1(v_1 c)\} R_{57} \\
 R_{75} &= \frac{H_1(v_0 c)}{J_1(v_1 c)} R_{12} \\
 R_{73} &= \{e_s V_{-1} Y_0(v_1 d) J_1(v_2 d) - e_a V_{-2} Y_1(v_1 d) J_1(v_2 d)\} \{Y_1(v_2 d) R_{59} - R_{60} J_1(v_2 d)\} \\
 &\quad + \{Y_1(v_2 d) e_a V_{-2} J_1(v_2 d) - e_a V_{-2} Y_0(v_2 d) J_1(v_2 d)\} \{R_{59} Y_1(v_1 d)\} \\
 R_{76} &= R_{73} - \frac{Y_1(v_1 c)}{J_1(v_1 c)} R_{72} \\
 R_{77} &= \frac{e_a V_0 H_0(v_0 c)}{e_s J_0(v_1 c)} R_{72} \\
 R_{78} &= R_{73} - R_{72} \frac{Y_0(v_1 c)}{J_0(v_1 c)}
 \end{aligned}$$

By varying various parameters occurring in expression of  $A_6$  one can get the characteristics of the radiation field.

### 3. RESULTS AND CONCLUSION

The value of  $|A_6|$  was computed with the help of IBM-1130 computer for various values of parameters given by  $e_a$  and  $e_s (\gamma_0 = 10^{-12}, \lambda_0 = 500 \cdot 10^{-8}, t = 0.1, k = \omega/c)$ . The computed results are given in table 1 and also plotted

Table 1. Effect of the radius of the ring source (a)  
 $a_1 = 0.1$  cm,  $b = 2.8$  cm,  $c = 2.80005$  cm,  $d = 2.800055$  cm,  
 $e = 2.800105$ ,  $\omega = 2\pi \cdot 10^{11}$  rad sec $^{-1}$ ,  $a = 2.0$  cm

The direction (in degrees) given below have less intense radiation field (in relative units). The directions other than these have the maximum amplitude of the radiation field (in relative units having value 1.11421356)

72.50, 15.25, 12.75, 7.75, 48.25, 14.25, 25.25, 24.0, 31.0, 18.0, 23.50, 75.5, 49.25, 47.5, 7.0, 44.0, 18.25, 17.25, 45.5, 36.75, 45.0, 59.5, 17.5, 2.0, 4.5, 2.25, 1.25, 1.0, 24.25, 2.5, 0.75, 5.0, 17.5, 0.5, 0.25, 73.74, 2.75, 14.75, 24.5, 42.75, 3.0, 4.5, 12.5, 43.0, 1.75, 31.50, 25.75, 4.25, 3.25, 37.25, 38.0, 16.0, 31.25, 30.25, 17.0, 76.25, 32.0, 19.25, 67.0, 54.25, 3.5, 4.0, 60.25, 37.5, 16.25, 41.5, 15.75, 3.75, 90.0, 30.50, 8.0, 48.5, 54.0, 19.0, 5.75, 54.75, 24.75, 25.0, 16.5, 23.75, 8.25, 42.0, 17.25, 18.75, 31.75, 26.0, 26.75, 11.5, 16.75

These directions have the magnitude of the radiation field respectively  
1.3923, 1.3535, 1.3425, 1.3050, 1.2806, 1.2719, 1.2627, 1.2114, 1.1960, 1.1799, 1.1778, 1.1655,  
1.1533, 1.1443, 1.1285, 1.1206, 1.1112, 1.0891, 1.0819, 1.0694, 1.0683, 1.0636, 1.0622, 1.0615,  
1.0586, 1.0547, 1.0525, 1.0456, 1.0451, 1.0424, 1.0395, 1.0376, 1.0366, 1.0349, 1.0320, 1.0284,  
1.0269, 1.0198, 1.0178, 1.0137, 1.0129, 1.0113, 1.0105, 1.0067, 1.063, 1.0049, 1.0042, 1.0036,  
1.0034, 1.0028, 1.0022, 1.0018, 1.0016, 1.0015, 1.0014, 1.0007, 1.0007, 1.0003, 1.0002, 1.0002,  
1.0001, 1.0, 1.0, 1.0, 0.9833, 0.9146, 0.8735, 0.8676, 0.6889, 0.6825, 0.6635, 0.6503, 0.5502,  
0.5193, 0.4945, 0.4022, 0.3173, 0.2302, 0.2213, 0.2029, 0.1353, 0.0986, 0.0918, 0.0798, 0.0743,  
0.0743, 0.0282, 0.0188, 0.0005.

in the fig. 2. The plot shows the effect of the radius of the central conductor on radiation pattern. For having more precision in direction the plot is made only for first 10 degrees. This is a fraction of the total radiation pattern but this surely exhibits the general behaviour of the radiation field. For  $a_1 = 0.1\text{ cm}$

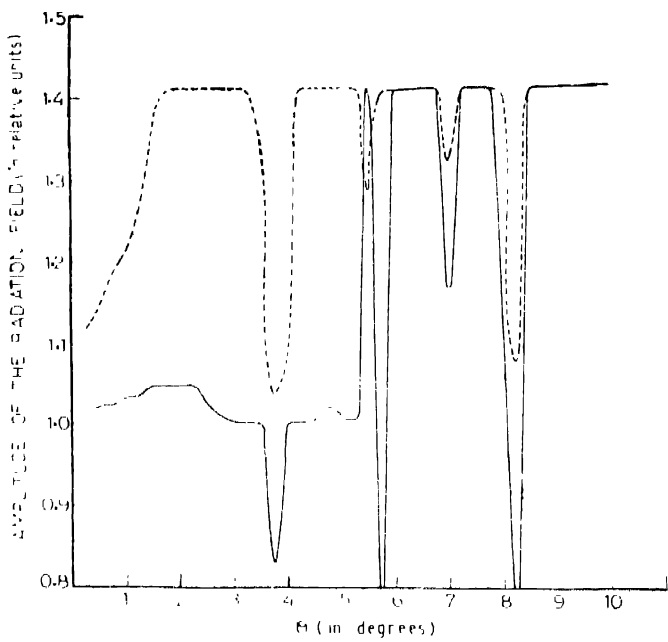


Fig. 2. Variation of the field amplitude with direction  $\theta$   
--- solid curve  $a_1 = 0.1\text{ cm}$ ,  
- - - - - dotted curve  $a_1 = 0.15\text{ cm}$   
 $\left\{ \begin{array}{l} a = 1.85\text{ cm}, \quad b = 2.8\text{ cm}, \quad c = 2.80005\text{ cm} \\ d = 2.800055\text{ cm}, \quad e = 2.800105\text{ cm}, \quad w = 2\pi \times 10^{12}\text{ rad/sec} \end{array} \right.$

(solid plot) it shows less broad maximum radiation zone while for  $a_1 = 0.15\text{ cm}$  (dotted line) there occurs five broad radiation zones having maximum radiation ( $1.75^\circ$  to  $3.25^\circ$ ,  $4.25^\circ$  to  $5.25^\circ$  to  $6.75^\circ$ ,  $7.25^\circ$  to  $8.0^\circ$  and beyond  $8.0^\circ$ ). This is of course a small range of direction but it is obvious when one proceeds towards higher  $\theta$  the maximum radiation zone gets broadened and from  $80^\circ$  to  $89.75^\circ$  (for  $a_1 = 0.1\text{ cm}$  and  $a_1 = 0.15\text{ cm}$ ) there occurs a uniform radiation zone having maximum radiation. But the direction of  $90^\circ$  remains to be having less intense radiation in all the cases. The increase in  $a$  causes the more directions having less intense field (table 1). The variation in pattern due to the variation in other parameters is not reported here. This structure essentially gives the broad multi-peaks while on excitation with the help of ring source. It is worth noticing that these results are obtained with the help of theoretical background and need to be verified experimentally. As far as the physical explanations are concerned the leaky waves excitation on such surface are the well proved phenomenon and

hence the explanation rendered by Tamir *et al* (1962, 1963) seems to be most promising and acceptable. This radiation system can be visualized as multi-layered multipole radiating system.

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